EXERCISE – V JEE PROBLEMS

1. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. **[JEE 2003, 4]**

2. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k equals **[JEE 2004(Scr.)]**
(A) 2/9 (B) 9/2 (C) 0 (D) –1

3. Let P be the plane passing through (1, 1, 1) and parallel to the lines L_1 and L_2 having direction ratios (1, 0, –1) and (–1, 1, 0) respectively. If A, B and C are the points at which P intersects the coordinate axes, find the volume of the tetrahedron whose vertices are A, B, C and the origin. **[JEE 2004, 2]**

4. (a) A variable plane at a distance of 1 unit from the origin cuts the co-ordiante axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the values of k is
(A) 3 (B) 1 (C) 1/3 (D) 9
[JEE 2005 (Scr.), 3]

(b) Find the equation of the plane containing the line $2x-y+z-3=0, 3x+y+z=5$ and at a distance of $1/\sqrt{6}$ from the point (2, 1, –1). **[JEE 2005 (Mains), 4]**

5.(a) A plane passes through (1, –2, 1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2z = 4. The distance of the plane from the point (1, 2, 2) is **[JEE 2006, 3]**
(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

(b) Match the following **[JEE 2006, 6]**

Column–I	Column–II
(A) Two rays in the first quadrant x + y = | a | and ax – y = 1 intersects each other in the interval $a \in (a_0, \infty)$, the value of a_0 is	(P) 2
(B) Point (α, β, γ) lies on the plane x + y + z = 2. Let $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$. $\hat{k}\times(\hat{k}\times\vec{a})=0$, then γ equal	(Q) 4/3
(C) $\left\|\int_0^1 (1-y^2)\,dy\right\|+\left\|\int_1^0 (y^2-1)\,dy\right\|$	(R) $\left\|\int_0^1 \sqrt{1-x}\,dx\right\|+\left\|\int_{-1}^0 \sqrt{1+x}\,dx\right\|$
(D) In a ΔABC, if sin A sin B sin C + cos A cos B = 1, then the value of sin C equal	(S) 1

(c) Match the following **[JEE 2006, 6]**

Column–I	Column–II
(A) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right)=t$, then tan t equal	(P) 0
(B) Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1=\frac{a}{b+c}$, $\cos\theta_2=\frac{b}{a+c}$, $\cos\theta_3=\frac{c}{a+b}$ then $\tan^2\frac{\theta_1}{2}+\tan^2\frac{\theta_3}{2}$ equal	(Q) 1
	(R) $\frac{\sqrt{5}}{3}$
(C) A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(S) 2/3

6.(a) Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5. **[JEE 2007, 3+6]**
Statement-I : The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.
because
Statement-II : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes.
(A) Statement-I is true, Statement-II is true; Statement-II is correct explanation for Statement-I
(B) Statement-I is true, Statement-II is true; Statement-II is **NOT** correct explanation for Statement-I
(C) Statement-I is true, Statement-II is False
(D) Statement-I is False, Statement-II is True

MATCH THE COLUMN
(b) Consider the following linear equations
ax + by + cz = 0
bx + cy + az = 0
cx + ay + bz = 0
Match the conditions/expressions in **Column–I** with statements in **Column–II.**

Column–I	Column–II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(P) the equation represent planes meeting only at a single point.
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(Q) the equation represent the line $x = y = z$
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(R) the equation represent identical planes
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(S) the equation represent the whole of the three dimensional space.

7.(a) Consider three planes **[JEE 2008, 3+4+4+4]**

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1 & P_1 and P_2 respectively.

Statement-I : At least two of the lines L_1, L_2 and L_3 are non-parallel.

because

Statement-II : The three planes do not have a common point.

(A) Statement-I is true, Statement-II is true; Statement-II is correct explanation for Statement-I

(B) Statement-I is true, Statement-II is true; Statement-II is **NOT** correct explanation for Statement-I

(C) Statement-I is true, Statement-II is False

(D) Statement-I is False, Statement-II is True

Paragraph for Question Nos. (i) to (iii)

(b) Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$;

$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

(i) The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

(ii) The shortest distance between L_1 and L_2 is

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

(iii) The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

8. (a) Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is **[JEE 2009, 3+3+4]**

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

(b) A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordiantes axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

(c) Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations $3x - y - z = 0$ & $-3x + z = 0$, $-3x + 2y + z = 0$ Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is

9. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is **[JEE 2010]**

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

10. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is **[JEE 2010]**

11. If the distance of the point P(1, –2, 1) from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$ is 5, then the foot of the perpendicular from P to the plane is **[JEE 2010]**

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

12. Match the statements in Column I with the values in Column II **[JEE 2010]**

Column–I	Column–II
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ & $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(P) – 4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(Q) 0
(C) Non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b}$ = 0, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c})$ = 0 and $2\lvert\vec{b}+\vec{c}\rvert = \lvert\vec{b}-\vec{a}\rvert$, If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(R) 4
(D) Let f be the function on $[-\pi,\pi]$ given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. Then the value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(S) 5
	(T) 6

13. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is **[JEE 2012]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

14. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **[JEE 2012]**

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

15. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are) **[JEE 2012]**

(A) y + 2z = –1 (B) y + z = –1

(C) y – z = –1 (D) y – 2z = –1